Nature and Reduction of Errors in Near-Earth Autonomous Satellite Navigation

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This paper studies the potential of autonomous satellite navigation through prediction of both spacecraft ephemeris and orientation of the Earth. Error sources identified and discussed quantitatively are: aerodynamics; direct and indirect solar radiation; gravity of the Earth, sun, and moon; motion of the rotational pole; rotation rate of the Earth; satellite clock; and the spacecraft tracking system. A device that has already been demonstrated in orbit can be used to effectively cancel the acceleration uncertainties due to atmosphere and solar radiation. Navigation error introduced by the remaining error sources are determined for two spacecraft altitudes (1000 and 150 km) and a prediction interval of one year. Anticipated improvements in modeling the geopotential, tracking system accuracy, and satellite timekeeping are also discussed. The results show that this concept of an autonomous satellite navigation system is suitable for a wide variety of satellite missions.

Introduction

AUTONOMOUS navigation for scientific and military satellites has the advantages of independence from ground facilities and the ability to process data and broadcast data products in near-real time. Requirements for Positional accuracy are mission dependent but typically range from a few meters to hundreds of meters. Different approaches have been proposed and studied using external and self-contained observations to provide autonomy. Examples of external data sources are: the global positioning system (GPS),[1] landmarks,[2] stars and the moon,[3] and the Tracking and Data Relay Satellite System (TDRSS).[4] Satellite-mounted gravity gradiometers have been shown to have the potential to provide a totally self-contained data source.[5] In these approaches, data are processed to provide an estimate of position and other system parameters, e.g., spacecraft attitude and sensor parameters.

In this paper a different approach is taken to achieve the benefits of autonomous navigation. It is shown that with state-of-the-art spacecraft design technology, long-term prediction of satellite ephemerides and the orientation of the Earth could satisfy a wide range of autonomous missions. Anticipated improvements in modeling the geopotential, tracking system accuracy and satellite timekeeping are demonstrated to provide a significant increase in navigation accuracy.

Background

Navigation of a near-Earth spacecraft can be partitioned into two components: the position of the spacecraft and the orientation of the Earth. Each can be defined in a frame of reference whose origin is fixed at the center of mass of the Earth and whose orientation is specified with respect to the stars. In the equations of motion for the spacecraft, the following effects need be addressed: gravitation of the Earth, sun, and moon; Earth atmosphere; solar radiation; and satellite timekeeping. In determining the orientation of the Earth it is necessary to consider polar motion, Earth rotation, and nutation and precession. All of these factors and their concomitant error sources are considered below.

Spacecraft Position

Extrapolating the position of a spacecraft is an initial-value problem that requires estimates of at least the initial position and velocity and a model of the force environment. Initial conditions are usually determined from terrestrial tracking data, such as range or range rate over the order of days. Errors in the initial conditions can arise from errors in tracking data and the model of the force field used to relate the observations made at different times over the tracking interval. The accuracies of terrestrial tracking systems, such as laser and radio-frequency Doppler, are on the order of centimeters. An estimate of the orbit position error as a function of time is given by the tracking accuracy divided by the square root of the number of observations and multiplied by the ratio of the prediction interval to the tracking interval.

The gravitational environment of a near-Earth satellite is a result of the sun, moon, and Earth. Direct forces induced by the sun and moon can be computed by treating the sources as point masses and are well defined and understood.[6] Indirect forces arise through tides, e.g. ocean, atmospheric, and Earth body tides. After some early confusion, the systematic effects of the three different types are now well understood[7] and consistent estimates of their parameters have been obtained. These effects need to be modeled in the orbit extrapolation because they induce important long-period perturbations; however, they should not contribute significant residual errors if treated properly.

Accelerations induced by the Earth are dominant and, in many cases, limit the orbit accuracy. Examples of orbit precision follow. The LAGEOS spacecraft is at an altitude of about 6000 km with terrestrial laser ranging as its primary source of tracking data. Position accuracy is about 1-m rms over a span of one month and 15-m rms over 32 months.[8] The primary error source over the 1-month space is attributed to uncertainties in the geopotential. Satellites of the Navy Navigation Satellite System are at an altitude of about 1000 km with tracking data being radio-frequency Doppler ground-based observations. Estimates of position accuracy over spans of one to two days are about 6 m, with the errors also primarily due to the geopotential.[9] The lower the altitude of the spacecraft, the larger the effects of errors in the geopotential models.

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Orbit determination and extrapolation for low-altitude spacecraft are adversely influenced by uncertainties in the atmospheric density. Forces from the Earth's atmosphere are significant because of their unpredictability and direction rather than magnitude. The significant aerodynamic force is drag which is antiparallel to the spacecraft velocity. An average coefficient of drag and area-to-mass ratio can be determined through precision orbit determination. Modeling errors result primarily from uncertainty in the atmospheric density and uncertainties in the coefficient of drag and area-to-mass ratio due to a lack of accurate attitude information. The atmospheric density varies nearly exponentially with altitude and is generally parameterized by solar radiation intensity, geomagnetic activity, latitude, time of the year, and local time.[10] Errors in estimating density arise from both a lack of accuracy of the models and uncertainties in predicting the solar radiation intensity and geomagnetic activity. Drag forces at altitudes between 900 and 1200 km have been studied extensively[11] and an estimate of 10% for the drag force uncertainty appears reasonable. At these altitudes, the drag force varies between 2 and $30\times10^{-10}g$ ($g=9.8$ m-s^{-2}) for the typical range of solar activity and area-to-mass ratios. The along-track position error due to an error in along-track force can be determined from the perturbed equations of orbital motion to be $1.5at^2$ where a is the specific force and t the elapsed time. As a result, the positional error is about 2-33 m/day^{-2}. Significantly larger errors can be expected at lower altitudes because the scale height at these altitudes is about 150 km. Uncertainty in the drag force is generally the largest source of error in ephemeris prediction for a low-altitude satellite.

Spacecraft motion is also affected by direct solar radiation and the Earth's albedo. Direct solar radiation has been treated by Cook[6] and the Earth's albedo, which has significantly less effect, by Slowey.[12] The force due to direct solar radiation is given by $(I/c)(A/M)S$ where I is the incident power, c the speed of light, A/M the area-to-mass ratio, and S a parameter that varies between 1 and 2 depending on the shapes and properties of the spacecraft surfaces. At an altitude between 600 and 700 km, direct solar radiation force is about equal to the drag force. Since direct solar radiation is primarily periodic, the effect on spacecraft position and velocity is significantly reduced. The accuracy to which it can be modeled depends primarily on the configuration of the spacecraft, its surface properties, and knowledge of the attitude.

The dominant error in position arising from spacecraft clock errors can be expressed as a product of the clock error and the velocity of the spacecraft relative to the Earth. Radiation-hardened crystal oscillators are used for satellite timekeeping in the Navy Navigation Satellite System. The better crystal oscillators have long-term drifts that are predictable to about 8×10^{-11} over one year and quadratic in nature.[13] The mean velocity of near-Earth satellites is weakly dependent on altitude, e.g.,7814 m-s^{-1} at 150 km vs 7252 m-s^{-1} at 1200 km. Using 7814 m-s^{-1}, the maximum anticipated position error at the end of one year would be about 20 m. Cesium standards for the GPS program, although not demonstrated, are projected to have an accuracy of about 7×10^{-12} over one year which is equivalent to a position accuracy of about 2 m.

Earth Orientation

To locate the position of a spacecraft with respect to the Earth's surface, it is necessary to define the orientation of a frame of reference fixed in the Earth with respect to the inertial or quasiinertial reference frame in which the spacecraft is defined. This transformation involves knowledge of the orientation of the instantaneous axis of rotation (polar motion), the variable rate of rotation, and the nutation and precession of the poles of the equator and the ecliptic. Nutation and precession are well known and can be determined from formulas.[14] Estimates for pole position and the variability of the rotation rate are available on a monthly basis because they are more difficult to extrapolate. Estimates of extrapolation errors are[15]:

$$S_p^2 = 0.012^2 + 0.00023^2 d^2$$

$$S_t^2 = 0.0031^2 + 0.00024^2 d^2$$

where S_p is the error in pole position,x or y (arcsec); S_t the error in UT1-UTC (s); and d the extent of extrapolation (days).

The satellite position errors depend on the inclination of the particular orbit of interest. The principal contribution to the errors, as seen in Fig. 1, is from the uncertainty in the rate of rotation. This error varies from a maximum at the equator to zero at the poles. Uncertainties in the orientation of the pole introduce errors in computing the orientation of the Earth and also in the orbit determination process. The induced ephemeris errors have a daily period and their effect on the initial orbit parameters can be minimized using fitting intervals that are significantly longer. This have been confirmed experimentally. In addition, it has been shown that the pole position can be determined as a part of the orbit determination procedure.[16]

Approach

In this section and approach to satellite autonomy is described that generates long-term predictions on the order of one year. The concept is an extension of that which has provided limited autonomy for the Navy Navigation Satellite System. The essential ingredient is a device called the Disturbance Compensation System (DISCOS) which eliminates the orbital effects of all surface forces. As a result, the major sources of prediction errors are eliminated, i.e., aerodynamic and direct and indirect solar radiation. This device was proposed nearly 20 years ago.[17] The system consists of essentially three elements: a sensor, a processor or controller, and a propulsion system. The sensor is a proof mass in a cavity shielded from the atmosphere and solar environments with a means to measure the proof mass position. The controller transforms the position data into signals that activate the propulsion system to avoid collisions between the proof mass and cavity wall. As a result, the spacecraft is forced to follow the gravitational orbit of the proof mass.

DISCOS was successfully demonstrated by a dedicated spacecraft experiment nearly 10 years ago.[18] An artist's concept of the spacecraft, called TRIAD, is given in Fig. 2. The specific mission was to demonstrate the concept in an 850-km-altitude polar orbit and to establish its utility for an improved spacecraft design for the Navy Navigation Satellite System. A detailed view of the integrated unit is given in Fig. 3, with a functional diagram given in Fig. 4. Position of the 22-mm-diam. gold-platinum alloy proof mass was measured by a capacitance bridge with resolution of 50μm. The relative displacement between the proof mass and cavity center was maintained at less than 1 mm. This system performed flaw-

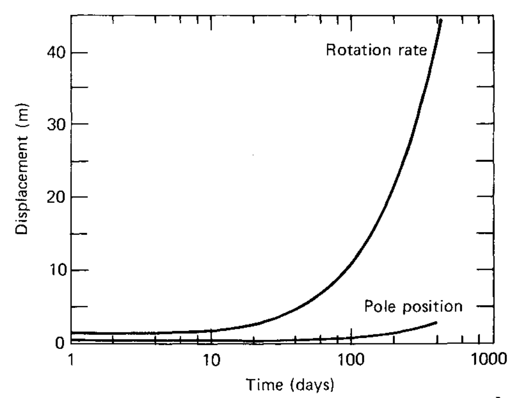

Fig. 1 Expected errors in extrapolating pole position and rotation rate.

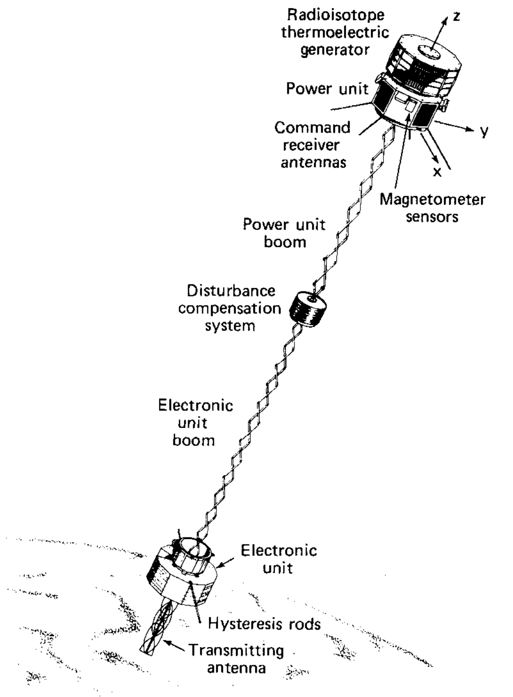

Fig. 2 TRIAD: advanced satellite navigation experiment.

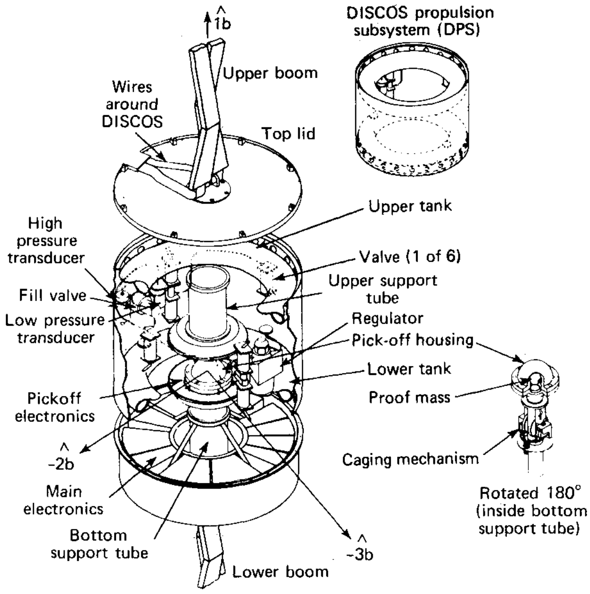

Fig. 3 Disturbance compensation system.

lessly for 14 months until the fuel of the cold gas propulsion system was exhausted some two months after its design life. Proof mass displacement for a 12-min interval during a period when the spacecraft was in the Earth's shadow is given in Fig. 5. Apparent are the segments of near-parabolic curves between thruster actions tht would be expected from atmospheric drag. The largest a priori source of orbit error was anticipated to be self gravity between the proof mass and spacecraft, but in-orbit tracking experiments confirmed that this acceleration was less than the 10^{-11} *g* design goal. An interesting aspect of the system, illustrated in Fig. 4, is an electrostatic means to induce a controlled force on the proof mass to compensate for the effects of self gravity and to be able to affect controlled changes to the ephemeris, i.e, provide a stationkeeping capability.

Subsequent to this successful navigation experiment, a single-axis version of the DISCOS concept was introduced in-

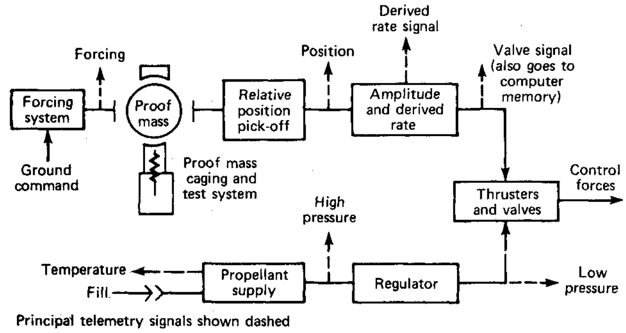

Fig. 4 Functional diagram of DISCOS control system.

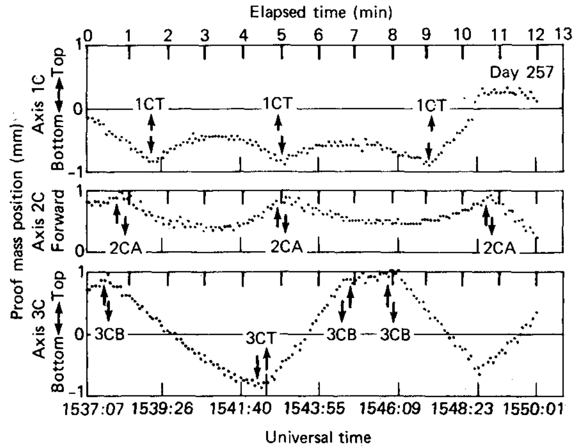

Fig. 5 DISCOS proof mass position.

to the second-generation spacecraft (NOVA) of the Navy Navigation Satellite System.[19] Most interesting is the use of a solid propellant teflon pulse-plasma thrusting system capable of producing a force of 15 μ·N/pulse that is designed for a 10-yr mission.[20,21] This propulsion system, including power conditioning and propellant, has a total mass of 6.4 kg-m.

Through the successful demonstration of the DISCOS concept on the TRIAD spacecraft in 1972 and the successful demonstration of a long-lived solid teflon pulse-plasma thrusting system on the NOVA spacecraft in 1981, the means to eliminate the orbital perturbations of atmospheric density and direct and indirect solar radiation have been demonstrated. As a result, the remaining primary sources of uncertainty in satellite navigation are the gravitational field, satellite clock, and orientation of the Earth. There is a broad base for continued improvements in our knowledge of the geopotential.[22] Global models for the geopotential became possible with the advent of artificial satellites. Earth-based tracking using precision radio and laser systems, spaceborne microwave altimetry, and in situ gravity observations have been used to generate models such as the GEM-9 that have up to about 600 parameters.[23] Accuracy of this model has been assessed from comparisons with 5-deg equal area terrestrial mean free-air anomalies at 9 m-gal rms.[24]

Several programs have been proposed to improve our knowledge of the geopotential. Near-term improvements are possible which should result in about a factor of 2 improvement on wavelengths as small as 2000 km.[25] This would involve a new solution using improved physical, mathematical, and statistical models, and the addition of existing data that has not yet been used. The proposed Geopotential Research Mission (GRM) should provide a more significant improvement. This satellite mission is to consist of two spacecraft in identical circular polar orbits but separated in distance by between 100 and 300 km at an altitude of about 160 km.[36] By making range-rate measurements between spacecraft to an ac-

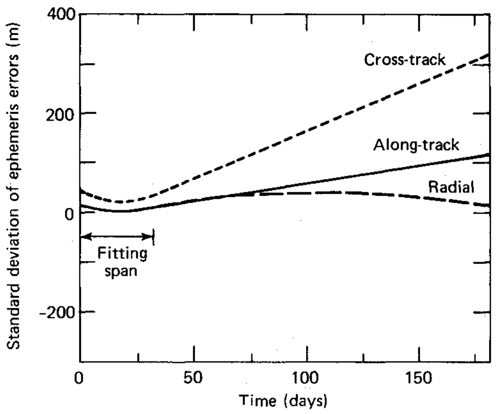

Fig. 6 Long-term ephemeris prediction errors: 1000-km altitude, 30-day fit, zonals not corrected, five-month prediction.

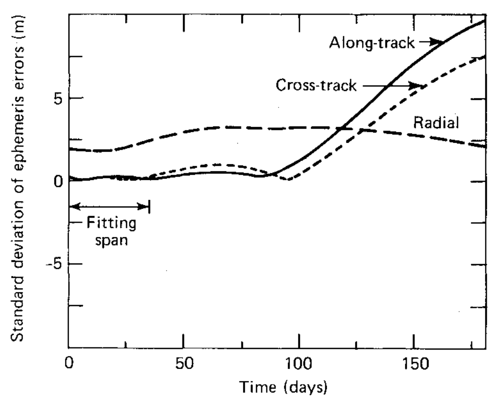

Fig. 7 Long-period ephemeris prediction errors: 1000-km altitude, 30-day fit, zonals corrected, five month prediction.

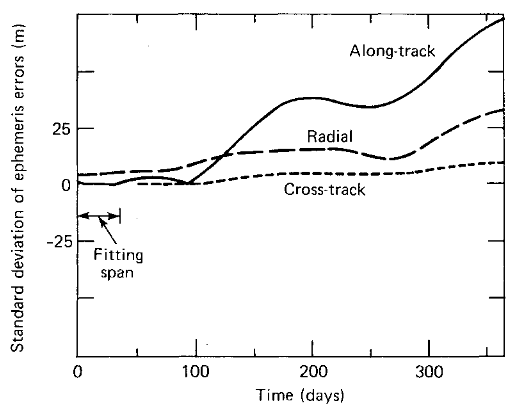

Fig.8 Long-period ephemeris prediction errors: 1000-km altitude, 30-day fit, one-year prediction.

curacy of 1 μm-s^{-1}, a factor of 20 to 200 improvement should be realized over our current knowledge of the geopotential.

Significant improvements are anticipated in the near-term in satellite timekeeping.[27] New types of crystals are being studied and are expected to result in improvements of about an order of magnitude.[28] Atomic standards under development for spacecraft application should also have significantly reduced long-term uncertainties.

No significant improvements have been forecast for predicting the orientation of the Earth. However, monitoring of both the pole position and variations in the rotation rate by very long baseline interferometry portend a better understanding and possibly an improvement in the predictive techniques.

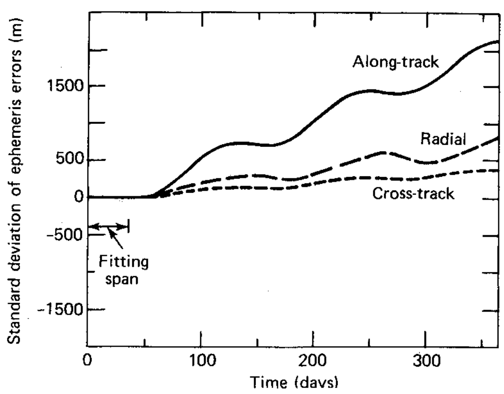

Fig. 9 Long-period ephemeris prediction errors: 150-km altitude, 30-day fit, one-year prediction.

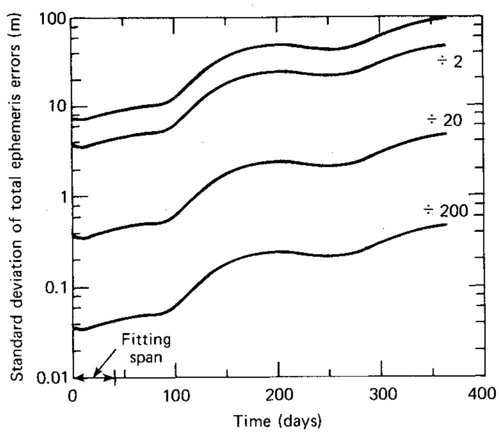

Fig. 10 Total ephemeris prediction errors at 1000 km—current and improved gravity models. The family of curves shows the improved prediction errors at 1000-km atltitude which will accompany expected improvements in the gravity model, assuming the presence of a DISCOS to remove drag and radiation effects.

Improvements in tracking system accuracy should improve modestly. New high-precision lasers are being developed with instrument measurement precision on the order of 2 cm.[27] Improvements in the knowledge of the propagation velocity and station location will result from ongoing programs.

The remainder of this paper is devoted to a quantitative estimate of the accuracy that can be realized in satellite navigation over periods of up to one year by orbit extrapolation for spacecraft incorporating a DISCOS device and taking into account the improvements that are likely to occur in the representation of the geopotential and the accuracy of spaceborne clocks and satellite tracking systems.

Earth Gravity Errors

With a DISCOS system, the errors in a predicted spacecraft ephemeris will be dominated by errors in the Earth gravity model. In this context, the sun and moon can be treated as point masses, and their parameters are known well enough that their effects can be computed to high accuracy (less than 1-m error) via higher order perturbation techniques.

Orbital effects of the Earth gravity field can be partitioned into two categories: 1) long-period variations and secular effects, and 2) short-period variations (i.e., periods less than one day). With the exception of sidereal resonance, the long-period and secular effects are due to the zonal harmonic terms or their equivalent in the gravity field expansion. Resonance with certain nonzonal terms occurs when the orbital period is nearly commensurate with an integer submultiple of the Earth sidereal rate. Where the orbit period is very close to an integer number of revolutions per sidereal day, the resonance effect can be quite large and long in period; however, except for this special case, resonance can be treated as a high-frequency effect. In this study, this special case has been ignored.

Errors in the Earth gravity model have both direct and indirect effects on the predicted satellite ephermeris. The direct effect results from integrating an erroneous force field. The indirect effect lies in the corruption of the fitted "initial-condition" parameters for integration. The most serious effect of this aliasing is on the period or semimajor axis parameter. A long-period along-track error can appear as a linear growth over a fitting span that is small relative to the period and would be absorbed into the orbital period parameter. A longer fitting span would reduce this initial-condition aliasing.

To study the effects of gravity model errors, an analytic orbit prediction program was used. This integrator was developed in 1970 for accurate long arc (> 1 year) orbit prediction. It uses a seminumerical technique to predict the behavior

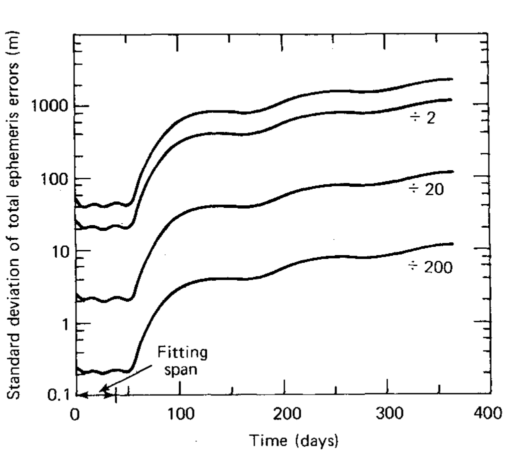

Fig. 11 Prediction errors at 150 km—current and improved gravity models. The family of curves shows the improved prediction errors at 150-km altitude which will accompany expected improvements in the gravity model, assuming the presence of a DISCOS to remove drag and radiation affects.

of mean orbital elements, and recovers osculating position and velocity using first-order analytic expressions for the short-period effects.[29] Long-period and secular rates of all of the perturbing forces except J_2 are computed from a first-order theory[30] for each orbital revolution. Second-order expressions are used to compute the secular rates due to J_2.[31] The secular rates of the mean elements are then numerically integrated with a time step of 1 rev. The mean elements are defined to be in "true equatorial system of date." Precession and nutation effects in this frame are treated as kinematical perturbations, and are included as a term in the long-period prediction.

A simulation study was made using the above program to assess the effects of gravity model errors on predictions. Drag and radiation pressure forces were eliminated to simulate the presence of a DISCOS device. The simulation runs were carried out by fitting initial conditions for an assumed Earth model to data generated from a second or truth model, assuming perfect position measurements over the fitting span. Extrapolations were then made to obtain prediction errors based on the truth model. Two cases were studied: 150- and 1000-km altitude, both at 68-deg inclination. Two of the latest geopotential models were used in the study, under the assumption that their differences are a representative estimate of the uncertainty of current models. The GEM-10 model[23] was used as the truth model, and the SAO-72[32] was used as the assumed model.

In these simulation runs only the long-period and secular terms were included, since these are the source of significant errors in long predictions. The short-period errors do not grow in time, but appear as noiselike oscillations superimposed on the long-period extrapolation errors. Short-span (one day) fits were performed with short-period terms included to determine the rms error arising from them. The nonzonal resonance terms were the largest component of these errors, with amplitude dependent on how close the period is to resonant conditions. For any given satellite, the resonance errors can be reduced significantly by adjusting the appropriate nonzonal model coefficients based on a span of observations. Under the assumption that resonance errors would be removed in this manner, they were eliminated from the simulation results. The residual rms short-period effects were observed to be 5 m at 1000-km altitude and 40-m rms at 150-km altitude. Since these

Table 1 Satellite navigation error for one-half year (m)

	1000-km altitude			150-km altitude		
	Current	Near-term	Long-term	Current	Near-term	Long-term
Satellite ephemeris						
Geopotential	40	20	2-0.2	1000	500	50-5
Satellite clock	5	3	—	5	3	—
Tracking	6	3	1	6	3	1
Earth orientation						
Rotation rate	20	20	20	20	20	20
Pole position	1	1	1	1	1	1
Total	45	29	21-20	1000	500	54-21

Table 2 Satellite navigation error for one-year prediction (m)

	1000-km altitude			150-km altitude		
	Current	Near-term	Long-term	Current	Near-term	Long-term
Satellite ephemeris						
Geopotential	100	50	5-0.5	2000	1000	100-10
Satellite clock	20	2	1	20	2	1
Tracking	12	6	1	12	6	1
Earth orientation						
Rotation rate	45	45	45	45	45	45
Pole position	3	3	3	3	3	3
Total	112	66	45	2001	1001	110-46

errors are superimposed on any long-period errors, they need to be added in an rss sense to determine the total error.

Figure 6 shows the result at 1000-km altitude of fitting initial conditions over a 30-day span, and then extrapolating the results for 150 days. The three curves show the standard deviation of the long-period position error components resolved in the local vertical frame: radial, along-track and cross-track.[33] The long-period rms position error was computed from the osculating position error history over one complete orbital revolution. The secular growth in the errors is the result of errors in the zonal harmonic parameters. By including a pair of zonal coefficients as part of the initial-condition solution, these extrapolation errors can be reduced significantly, as shown in Fig. 7. This is a repeat of the case shown in Fig. 6, with a correction fitted to J_2 and J_3, the first even and odd zonal coefficients. It is assumed subsequently that any attempt to fit initial conditions to be used in a prediction of one year or more would necessarily include a similar adhoc correction of the gravity model to improve the accuracy.

The case illustrated in Fig. 7 was repeated with the fitting span reduced to 15 days from 30, and no significant difference was found in the prediction errors. In the real-world case, with intermittent coverage and short-period geodesy errors present, it may be necessary to lengthen the fitting span somewhat to achieve comparable results. However, a 30-day span with corrections fitted to at least two zonal coefficients appears to be near optimum, therefore, this is the method used here.

Figures 8 and 9 show the one-year long-period prediction errors in the mean elements based on a 30-day fitting span with zonal corrections applied for altitudes of 1000 and 150 km, respectively.

The family of curves shown in Figs. 10 and 11 are the rss of the short-period errors with the long-period errors shown in Figs. 8 and 9. Also shown are the results expected for the various factors improvement discussed earlier—a factor of 2 expected near-term, and a factor of 20 to 200 expected from the NASA-proposed Geopotential Research Mission.

Navigation Errors

Navigation error is defined here as the rss of the satellite position error and the Earth orientation error. With the DISCOS system effectively cancelling the effects of atmospheric and direct and indirect solar radiation forces, the remaining error sources are: the geopotential, satellite clock, tracking system, Earth rotation rate, and position of the rotational pole. The combined effects of these errors have been evaluated for satellites at two altitudes (1000 and 150 km) and a nominal inclination of 68 deg. Results are tabulated in Tables 1 and 2 for intervals of one-half and one year respectively. The columns titled current are based on the current state-of-the-art. The columns entitled near-term and long-term are based on improvements expected in 2-5 and 5-10 yr, respectively, and were discussed under "Approach."

Summary

This study demonstrates that autonomous satellite navigation is possible through long-range prediction of the spacecraft ephemeris and the orientation of the Earth. The error sources in the approach to autonomy that are discussed are: aerodynamics; direct and indirect solar radiation; gravity of the Earth, sun, and moon; motion of the rotational pole; satellite clock; rotation rate of the Earth; and the spacecraft tracking system. Elimination of the aerodynamic and solar radiation acceleration uncertainties can be accomplished through the DISCOS device, which is described, along with in-orbit test results. The remaining sources of errors are studied for two specific spacecraft altitudes, 1000 and 150 km. Autonomous navigation errors are given for prediction spans of one-half and one full year. Estimates have been made for the reduction of the magnitudes of the errors in the near-term (2-5 yr) and the long-term (5-10 yr). Results based on these estimates are also given.

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